

Theory of cooling neutron stars versus observations

D. G. Yakovlev^{*,†}, O. Y. Gnedin^{**}, A. D. Kaminker^{*} and A. Y. Potekhin^{*,‡}

^{*}*Ioffe Physico-Technical Institute, Politeknicheskaya 26, 194021, St. Petersburg, Russia*

[†]*Joint Institute for Nuclear Astrophysics, 210a Nieuwland Science Hall, Notre Dame, IN 46556-5670, USA*

^{**}*University of Michigan, 500 Church Street, Ann Arbor, MI 48109-1042, USA*

[‡]*Ecole Normale Supérieure de Lyon, CRAL (UMR CNRS 5574), 46 allée d'Italie, Lyon 69364, France*

Abstract. We review current state of neutron star cooling theory and discuss the prospects to constrain the equation of state, neutrino emission and superfluid properties of neutron star cores by comparing the cooling theory with observations of thermal radiation from isolated neutron stars.

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INTRODUCTION

The equation of state (EOS) of superdense matter in neutron star cores is still a mystery. It is not clear if it is soft, moderate or stiff; if the matter contains nucleons/hyperons, or exotic components. In the absence of good practical theory of supranuclear matter the problem cannot be solved on purely theoretical basis, but it can be solved by comparing theoretical models with observations of neutron stars. The attempts to solve this long-standing problem by different methods are numerous (e.g., Refs. [1, 2]). Here we discuss current results obtained from studies of cooling isolated neutron stars.

The first papers on neutron star cooling appeared with an advent of X-ray astronomy, before the discovery of neutron stars. Their authors tried to prove that neutron stars cool not too fast and can be discovered as sources of thermal surface X-ray radiation. The first estimates of thermal emission from cooling neutron stars were most probably done by Stabler [3] in 1960. Four years later Chiu [4] made similar estimates and analyzed the possibility to discover neutron stars from their thermal emission. First, simplified calculations of neutron star cooling were done in 1964 and 1965 [5, 6, 7]. The foundation of the strict cooling theory was laid in 1966 by Tsuruta and Cameron [8], one year before the discovery of pulsars. We review the current state of the cooling theory. More details can be found in recent review papers [9, 10].

COOLING NEUTRON STARS

Neutron stars are thought to consist of a thin crust and a massive core (e.g., Ref. [1]). The crust constitutes a

few per cent of the star's mass and is about ~ 1 km thick. The mass density ρ at the crust-core boundary is $\approx \rho_0/2$, where $\rho_0 \approx 2.8 \times 10^{14}$ g cm $^{-3}$ is the density of saturated nuclear matter. The crust is further divided into the outer and inner crust. The outer crust (more generally, the outer envelope; where ρ is below the neutron drip density $\lesssim 4 \times 10^{11}$ g cm $^{-3}$) consists of atomic nuclei and strongly degenerate electrons. In the inner crust, free neutrons appear in addition to the electrons and nuclei; these neutrons can be in superfluid state.

The neutron star core can be divided into the outer and inner core. The outer core extends to $\rho \sim (2-3)\rho_0$ and consists of neutrons (n), with an admixture of protons (p), electrons (e) and possibly muons. All these particles are strongly degenerate. The inner core extends to the stellar center [to $\sim (10-20)\rho_0$ in most massive stars]. Its composition is very uncertain. It may be the same as in the outer core or essentially different. In particular, hyperons may appear there in addition to nucleons. Another possibility is the appearance of exotic matter (pion or kaon condensates or strange quark matter or mixed phases, as reviewed, e.g., in [1]). Nucleons, hyperons and quarks can be in superfluid state. Physical properties of matter at $\rho \lesssim \rho_0$ are more or less restricted by nuclear physics data, but at higher ρ they are uncertain. Typical baryon chemical potentials ~ 500 MeV in a neutron star core are most difficult for rigorous microscopic calculations. Note a lively discussion in the literature that compact stars (or some of them) can be not neutron stars but strange stars (e.g., Refs. [1, 2, 11]) built entirely or almost entirely of strange quark matter.

Neutron stars are born in supernova explosions with high internal temperature $T \sim 10^{11}$ K, but gradually cool down. In ~ 30 s after the birth a star becomes transparent for neutrinos generated in its interiors and transforms from a proto-neutron star (e.g., [12]) to an ordinary

TABLE 1. Main slow neutrino emission in nucleon matter

Process			Q_s [erg cm ⁻³ s ⁻¹]
Modified Urca	$n + N \rightarrow p + N + e + \bar{\nu}$	$p + N + e \rightarrow n + N + \nu$	$10^{20} - 3 \times 10^{21}$
Bremsstrahlung	$N + N \rightarrow N + N + \nu + \bar{\nu}$		$10^{19} - 10^{20}$

neutron star whose EOS is almost temperature independent (except near the very surface). At the later neutrino-transparent stage the star cools via neutrino emission from its interior and via heat transport to the surface and subsequent photon thermal emission.

Neutron star cooling is described by general relativistic equations [13] of heat diffusion inside the star with neutrino energy sources and surface photon emission. The solution gives the distribution of the temperature T inside the star versus time t , and the effective surface temperature $T_s(t)$. To simplify computations, one divides [14] the neutron star into the interior region and the outermost heat-blanketing envelope [extended to densities $\rho \leq \rho_b \sim (10^{10} - 10^{11})$ g cm⁻³]. The thermal structure of the blanketing envelope is studied separately in the stationary, plane-parallel approximation, that relates T_s to the temperature T_b at $\rho = \rho_b$. The diffusion equations are then solved in the interior ($\rho \geq \rho_b$).

The thermal photon luminosity of the star is $L_\gamma = 4\pi\sigma R^2 T_s^4(t)$, where R is the circumferential stellar radius. Both, L_γ and T_s , refer to a locally-flat reference frame on the stellar surface. A distant observer detects the “apparent” luminosity $L_\gamma^\infty = L_\gamma(1 - r_g/R)$ and the “apparent” effective temperature $T_s^\infty = T_s \sqrt{1 - r_g/R}$, where $r_g = 2GM/c^2$ is the Schwarzschild radius and M is the gravitational neutron star mass. If the surface temperature distribution is anisotropic (for instance, owing to a strong magnetic field), L_γ^∞ is determined by a properly averaged surface temperature (e.g., Ref. [15]).

The theory gives cooling curves, $T_s^\infty(t)$, and predicts three main cooling stages. The initial stage of internal thermal relaxation lasts $t \lesssim 10 - 50$ yr. At this stage, the neutron star crust stays hotter than the core and thermally decoupled from it (because of much stronger neutrino cooling in the core). The surface temperature is then insensitive to the physics of the core but strongly depends on physical properties of the crust [16, 17]. Since no neutron star has been observed at this stage, we will not discuss it in detail. The next stage of neutrino cooling with isothermal interior lasts $t \lesssim 10^5 - 10^6$ yr. At this stage the neutrino luminosity $L_\nu \gg L_\gamma$; thermal conduction is high and makes the stellar interior isothermal, with the main temperature gradient located in the heat blanketing envelope. The cooling is mostly regulated by a strong neutrino emission from the core. The surface temperature responds to the core cooling and depends on properties of superdense core. All cooling isolated neutron stars whose

thermal surface radiation has been detected seem to be at this neutrino cooling stage (or its very end). Therefore, we focus our attention on this stage. At the next photon cooling stage the star is cold; its neutrino emission dies out ($L_\nu \ll L_\gamma$) and the cooling is governed by photon surface emission.

Soon after the cooling starts (in minutes to a year, depending on the internal structure), the core temperature drops to $T \sim 10^9$ K. For this T , the internal thermal energy of the star is $\sim 10^{48}$ erg. The cooling theory shows how this heat emerges from the star.

NEUTRINO EMISSION MECHANISMS

Let us summarize the main neutrino emission mechanisms in the neutron star core. They are strongly affected by baryon superfluidity. More details can be found, e.g., in Refs. [18, 19, 20].

Neutrino emission in nonsuperfluid cores. The major neutrino mechanisms in nucleon matter of the outer core ($\rho \lesssim 2\rho_0$) are modified Urca process and nucleon-nucleon bremsstrahlung. They are listed in Table 1 (from Ref. [9]), where N denotes a nucleon (n or p). They are relatively weak and produce slow neutrino cooling. The modified Urca process differs from its direct Urca progenitor, described below, by an additional nucleon-spectator that is required to satisfy momentum conservation of reacting particles. All reactions involving electrons can involve muons instead (if available in dense matter).

At higher ρ , in the inner core, neutrino emission can be strongly enhanced by new mechanisms (Table 2, from [9]). The enhancement level greatly depends on the EOS and composition of superdense matter that is most important for the cooling problem. The strongest enhancement is provided by direct Urca process [21, 22] in nucleon (or nucleon-hyperon) matter. It is a sequence of two reactions (a beta decay and beta capture) forbidden in the outer core by momentum conservation. It is allowed in the inner core for the matter with rather high ($\gtrsim 11$ –13%) proton fraction (for EOSs with large symmetry energy of nuclear matter). If it is forbidden but the matter contains pion condensate, neutrino emission is enhanced (although weaker) by direct-Urca-type reactions involving quasinucleons \tilde{N} (superpositions of n and p) in pion-condensed matter. If pion condensate is absent,

TABLE 2. Leading processes of fast neutrino emission in nucleon matter and three models of exotic matter

Model	Process		Q_f [erg cm ⁻³ s ⁻¹]
Nucleon matter	$n \rightarrow p + e + \bar{\nu}$	$p + e \rightarrow n + \nu$	$10^{26} - 3 \times 10^{27}$
Pion condensate	$\tilde{N} \rightarrow \tilde{N} + e + \bar{\nu}$	$\tilde{N} + e \rightarrow \tilde{N} + \nu$	$10^{23} - 10^{26}$
Kaon condensate	$\tilde{B} \rightarrow \tilde{B} + e + \bar{\nu}$	$\tilde{B} + e \rightarrow \tilde{B} + \nu$	$10^{23} - 10^{24}$
Quark matter	$d \rightarrow u + e + \bar{\nu}$	$u + e \rightarrow d + \nu$	$10^{23} - 10^{24}$

but kaon condensate available, neutrino emission is enhanced (even more weaker) by direct-Urca-like process involving baryonic quasiparticles \tilde{B} in kaon condensed matter. Nearly the same enhancement is expected due to the direct Urca process involving d and u quarks in quark matter. The processes in the inner core (Table 2) can amplify the neutrino emission by 2–7 orders of magnitude and lead to fast cooling. It is also possible that the neutrino emission in the inner core is not enhanced.

The emissivity $Q_\nu(\rho, T)$ of slow and fast neutrino processes in nonsuperfluid matter can be written as

$$Q_{\text{slow}} = Q_s T_9^8, \quad Q_{\text{fast}} = Q_f T_9^6, \quad (1)$$

where $T_9 = T/(10^9 \text{ K})$, while Q_s and Q_f are slowly varying functions of ρ (Tables 1 and 2). Thus, the neutrino luminosities of massive and low-mass stars (with and without inner core) can be very different. For instance, in a young massive star, where the direct Urca is open and the core temperature is $T = 10^9 \text{ K}$, L_ν is as huge as $\sim 10^{46} \text{ erg s}^{-1}$. In a low-mass star at the same T , L_ν would be ~ 7 orders of magnitude lower. In both cases L_ν rapidly decreases when the star cools. For instance, one has $L_\nu \sim 10^{34} \text{ erg s}^{-1}$ for a low-mass star at $t \sim 10 \text{ kyr}$.

Neutrino emission in superfluid cores. Nucleons, hyperons, and quarks in dense matter can be in superfluid state (e.g., [23], also see [9, 19] for references). This superfluidity occurs via Cooper pairing of particles owing to an attractive component of their interaction (with the appearance of a gap in the particle energy spectrum near the Fermi level). Systematic simulations of cooling superfluid neutron stars were triggered by a remarkable paper of Page and Applegate [24]. Superfluidity of any particles is characterized by its own density dependent critical temperature $T_c(\rho)$. Microscopic calculations of $T_c(\rho)$ are extremely model dependent and give a large scatter of critical temperatures. Let us mention several general features.

Neutron pairing in the spin singlet state with zero angular momentum (1S_0) occurs in the inner neutron star crust (for free neutrons) and dies out in the core because singlet-state nuclear attraction turns into repulsion near the crust-core interface. Neutron pairing in the core may occur in a triplet state with unit angular momentum, 3P_2 (coupled to a spin triplet state with three units

of angular momentum, 3F_2). Cooper pairing of other particles in the core can occur either in a singlet state or in a triple state. Baryon superfluidity is affected by the presence of pion or kaon condensate (e.g., [25] and references therein). Calculated values $T_c(\rho)$ range from $\sim 10^8 \text{ K}$ to a few $\times 10^{10} \text{ K}$ and vanish at supranuclear densities (where attractive pairing interaction becomes inefficient).

A special case is presented by color superconductivity [26] owing to very strong pairing of unlike quarks in quark matter, where $T_c(\rho)$ can be as high as $\sim 5 \times 10^{11} \text{ K}$.

Any baryon superfluidity reduces neutrino processes involving these baryons (Tables 1 and 2) due to a gap in the baryon energy spectrum. At $T \ll T_c$, the reduction is exponentially strong. For instance, the powerful direct Urca process can be formally open in the inner core, but completely suppressed by a strong superfluidity of neutrons or protons.

In addition, when T falls below T_c , superfluidity initiates a specific neutrino emission due to Cooper pairing of baryons [27] which enhances neutrino cooling. This mechanism can enhance the neutrino luminosity of the star by a factor of $\lesssim 30 - 100$ over the modified Urca level [28, 29].

Superfluidity affects also neutron star heat capacity [19] and thermal conductivity [30, 31] but these effects are less strong.

OBSERVATIONS

We will compare cooling theory with observations of surface thermal radiation of isolated neutron stars. The current state of the observations is discussed, e.g., in Refs. [32, 33, 34]. In Table 3 we list 16 isolated neutron stars whose effective surface temperatures have been measured or constrained. For brevity, the stars are numbered, and these numbers are used in the text and figures.

The data include two neutron stars (1 and 2, the pulsars Crab and J0205+6449) in historical supernova remnants (SNRs); the famous Vela pulsar (7) and the similar PSR B1706–44 (8); the “pulsar-twins” J0538+2817 and B2334+61 (9 and 10); the “three musketeers” [Geminga, PSR B1055–52, and PSR B0656+14 (12, 14, and 11)]; one compact central object in SNR (RX J0822–4300,

TABLE 3. Observational limits on surface temperatures of isolated neutron stars

Number	Source	t [kyr]	T_s^∞ [MK]	Confid.	Model	Ref.
1	PSR B0531+21 (Crab)	1	<2.0	99.8%	BB	[37]
2	PSR J0205+6449 (in 3C 58)	0.82–5.4	<1.02	99.8%	BB	[38]
3	PSR J1119–6127	~ 1.6	≈ 1.2	–	mHA	[32]
4	RX J0822–4300 (in Pup A)	2–5	1.6–1.9	90%	HA	[39]
5	PSR J1357–6429	~ 7.3	≈ 0.766	–	mHA	[40]
6	RX J0007.0+7303 (in CTA 1)	10–30	<0.66	–	BB	[41]
7	PSR B0833–45 (Vela)	11–25	0.68 ± 0.03	68%	mHA	[42]
8	PSR B1706–44	~ 17	$0.82^{+0.01}_{-0.34}$	68%	mHA	[43]
9	PSR J0538+2817	30 ± 4	~ 0.87	–	mHA	[44]
10	PSR B2334+61	~ 41	~ 0.69	–	mHA	[32]
11	PSR B0656+14	~ 110	0.91 ± 0.05	90%	BB	[45]
12	PSR B0633+1748 (Geminga)	~ 340	~ 0.5	–	BB	[46]
13	RX J1856.4–3754	~ 500	0.434 ± 0.003	68%	mHA*	[47]
14	PSR B1055–52	~ 540	~ 0.75	–	BB	[48]
15	PSR J2043+2740	~ 1200	~ 0.44	–	mHA	[32]
16	RX J0720.4–3125	~ 1300	~ 0.51	–	HA*	[49]

source 4); the compact source RX J0007.0+7303 at the center of the SNR CTA 1 (6); two “dim” (“truly isolated”) neutron stars RX J1856.4–3754 and RX J0720.4–3125 (13 and 16); two young and energetic pulsars J1119–6127 and J1357–6429 (3 and 5); and the old pulsar J2043+2740 (15).

The ages and effective surface temperatures of many sources are rather uncertain. For the sources mentioned in [29, 35, 36] the choice of t and T_s^∞ is mainly the same as in these references. The ages of other sources are pulsar spindown ages, and the errorbars of t and T_s^∞ are chosen in the same way as in [29, 35, 36]. We have enlarged a possible age range of the source 2 to 5.4 kyr, the pulsar spindown age, because this source can be accidentally projected onto the SNR 3C 58 (Yu.A. Shibano, private communication, 2007). The values of T_s^∞ are inferred from observations (reported in the indicated references) assuming either black-body (BB) spectrum or a hydrogen atmosphere model (magnetic or nonmagnetic one, mHA or HA). For the sources 13 and 16, T_s^∞ is determined [47, 49] using models of hydrogen atmospheres of finite depth (mHA* and HA*, respectively). One important source, 1E 1207.4–5209, is not included in the table because of the problems to interpret its spectrum (although modeling of such spectra is progressing [50]).

BASIC COOLING CURVES

For illustration, we will mainly use models of neutron stars whose cores contain neutrons, protons and electrons and have a stiff phenomenological EOS proposed in Ref. [51] (model I for the symmetry energy and the

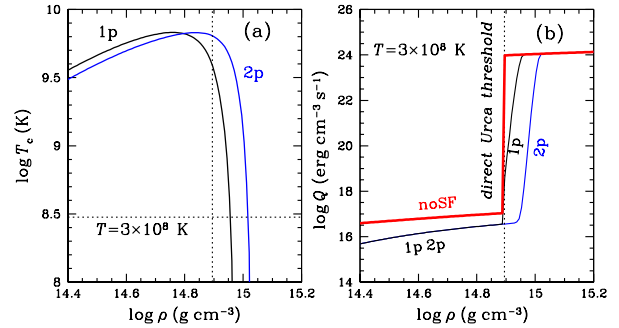


FIGURE 1. (a) Superfluid transition temperature versus density for two models (1p and 2p) of proton superfluidity in the neutron star core. (b) Neutrino emissivity profiles in the core at $T = 3 \times 10^8$ K for nonsuperfluid matter (noSF) and for matter with superfluid protons (models 1p or 2p).

bulk energy model that gives the compression modulus of saturated nuclear matter $K = 240$ MeV). The most massive stable neutron star, for this EOS, has the gravitational mass $M_{\max} = 1.977 M_\odot$ and the central density $\rho_c = 2.578 \times 10^{15}$ g cm⁻³; the direct Urca process is allowed at $\rho \geq \rho_D = 7.851 \times 10^{14}$ g cm⁻³ (in stars with $M \geq M_D = 1.358 M_\odot$).

Neutrino emissivity profiles in a stellar core at $T = 3 \times 10^8$ K are shown in Fig. 1b. The vertical dotted line shows the direct Urca threshold, ρ_D . The thick line (NoSF) is for nonsuperfluid matter. At $\rho < \rho_D$ the neutrino emission is determined by the modified Urca process. When ρ increases above ρ_D , the emissivity jumps by 7 orders of magnitude due to the direct Urca onset.

Fig. 2a presents the cooling curve for a $1.3 M_\odot$ non-superfluid star compared with the data. The curve is a

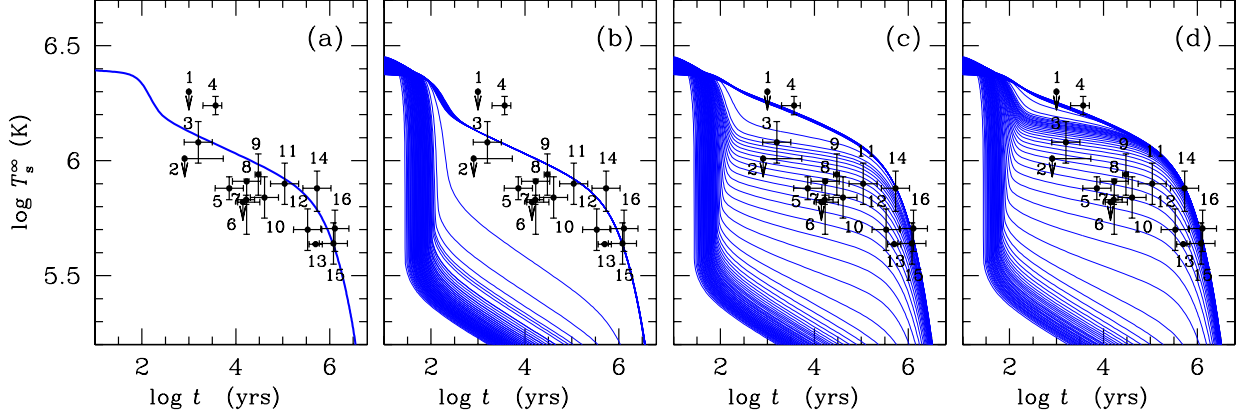


FIGURE 2. Observational limits on surface temperatures of isolated neutron stars compared to theoretical cooling curves. (a) Cooling curve for a nonsuperfluid $1.3M_{\odot}$ star. Any other panel shows 88 curves from $M = 1.1$ to $1.97M_{\odot}$ with step $0.01M_{\odot}$: (b) No superfluidity, sharp direct Urca threshold; (c) proton superfluidity 1p and (d) 2p which broaden the threshold.

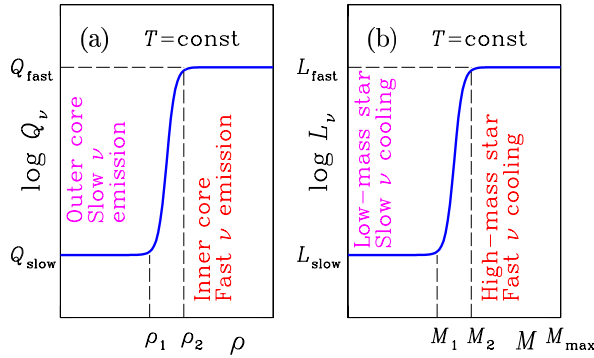


FIGURE 3. Very schematic view of (a) neutrino emissivity profile in a neutron star core and (b) neutrino luminosity versus star's mass at a fixed core temperature T . Units are arbitrary.

typical example of slow cooling via the modified Urca process. It is actually universal, being almost the same for all stars with $1.1M_{\odot} \leq M < M_D$, and for a wide class of EOSs [24]. This universal curve goes through the scatter of observational points but cannot explain the sources which are hottest and coolest for their age. These sources seem to have neutrino luminosities lower and higher than the modified Urca level.

Now consider a set of nonsuperfluid neutron star models of masses from 1.1 to $1.97M_{\odot}$, with the mass step of $0.01M_{\odot}$ (Fig. 2b). The first 26 models ($M \leq 1.35M_{\odot} < M_D$) belong to the class of low-mass stars and show the universal slow cooling behavior. The next $1.36M_{\odot}$ star has a small central kernel with the enhanced neutrino emission and shows much faster cooling (is much colder than all observed sources). All other more massive stars cool even faster via the direct Urca process and belong to the family of cold massive rapidly cooling stars. Evidently, these cooling curves cannot explain the data.

BASIC CONCEPT

A majority of realistic cooling scenarios, constructed up to now, satisfy the following basic phenomenological concept:

- (1) Low-mass stars undergo slow neutrino cooling.
- (2) Massive stars cool much faster via neutrino emission from their inner cores.
- (3) There is a family of medium-mass stars whose cooling is intermediate between the slow and fast ones.

Accordingly, the density profile of the neutrino emissivity in a neutron star core can look like that plotted in Fig. 3a; ρ_1 and ρ_2 mark the density range, where the slow cooling transforms into the fast one. This $Q_v(\rho)$ profile translates into a characteristic dependence of the neutrino luminosity L_v on the stellar mass shown in Fig. 3b; the transition between the slow cooling to the fast one takes place in the mass range from M_1 to M_2 .

Thus, at the present stage of investigation, the cooling theory *has potential to test four main parameters* of the neutrino emissivity function $Q_v(\rho)$ or the luminosity function $L_v(M)$. They specify the lower and upper levels of neutrino emission (either Q_{slow} and Q_{fast} or L_{slow} and L_{fast}) and the position of the transition zone between the slow and fast emission (either ρ_1 and ρ_2 or M_1 and M_2).

EXAMPLES OF PHYSICAL MODELS

Two examples [52] of successful realization of the above scheme are given in Figs. 2c and d. The figures show the cooling curves for neutron stars of $M = (1.1 - 1.97)M_{\odot}$ with strong proton superfluidity (1p or 2p) in the core (but with normal neutrons). The density profiles $T_c(\rho)$ of the proton critical temperature are shown in Fig. 1a; they are phenomenological and can be regarded as illustra-

tive. In the outer core, proton superfluidity is strong and suppresses the modified Urca process. Accordingly, the slow neutrino emission of low-mass stars is determined by weaker neutrino bremsstrahlung in neutron-neutron collisions. This rises the surface temperature of low-mass stars and allows one to explain observations of hottest sources.

In addition, proton superfluidity penetrates into the inner core ($\rho > \rho_D$) and suppresses the direct Urca process within the penetration depth. It broadens the direct Urca threshold and realizes a smooth transition from slowly to rapidly cooling neutron stars with the growth of M . Superfluidity 2p penetrates deeper than 1p which widens the range of masses of medium-mass stars. One can now explain observations of all the sources, including the coldest ones. Moreover, these scenarios predict the existence of very cold massive neutron stars (which have not been observed so far).

If the EOS and the critical temperature $T_c(\rho)$ in the stellar core were known, one would be able to “weigh” neutron stars [52]. For instance, the mass of the Vela pulsar would be $1.47 M_\odot$ for superfluidity 1p and $1.61 M_\odot$ for 2p. Unfortunately, neither EOS nor $T_c(\rho)$ are known and the “weighing” procedure is ambiguous. Nevertheless, as a rule, the ambiguity does not destroy the mass ordering of cooling stars. For instance, the Vela pulsar is expected to be more massive than RX J0822–4300. Moreover, one can invert the scheme and state that had the mass of one or several cooling neutron stars been measured, one could constrain the EOS and $T_c(\rho)$.

One can construct many other models of cooling neutron stars consistent with the data. One can take other EOSs in the core [with other direct Urca thresholds ρ_D and functions $T_c(\rho)$]. For instance, one can assume the presence of strong neutron superfluidity and normal protons (because cooling curves are nearly symmetric [53] with respect to exchanging neutron and proton $T_c(\rho)$ profiles).

The direct Urca threshold can also be broadened by nuclear effects (e.g., of pion polarization; see [54] and references therein) or by superstrong magnetic fields $\gtrsim 3 \times 10^{15}$ G [55] in a nonsuperfluid neutron star core.

LEVELS OF SLOW AND FAST NEUTRINO EMISSION

After several examples we can turn to a general analysis.

One can show that it is sufficient to lower the neutrino luminosity due to the modified Urca process in a low-mass star by a factor of $\sim 30 - 100$ to explain observations of stars hottest for their age. This low neutrino luminosity can be produced by nucleon-nucleon bremsstrahlung. The modified Urca process can be re-

duced by nucleon superfluidity, as in the above examples (Figs. 2c and d).

The level of fast neutrino emission in high-mass stars is less certain. To analyze it, let us consider (Fig. 4) three typical levels appropriate to the direct Urca process in nucleon/hyperon matter; to pion-condensed matter; and kaon-condensed matter. A schematic picture of the neutrino emissivity profiles through the stellar core for these scenarios is presented in Fig. 4a.

Fig. 4b shows the ranges of T_s^∞ (hatched regions) which can be explained by the theory assuming the scenario with the direct Urca process. The upper curve shows cooling of low-mass stars via neutron-neutron bremsstrahlung neutrino emission (as in Figs. 2c and d). The lower curve presents fast cooling of a maximum-mass star with direct Urca process open in the inner core. Any value of T_s^∞ between these two curves can be explained by a cooling of a star with some intermediate mass. As discussed above, this scenario explains all the data and predicts the existence of very cold neutron stars.

Fig. 4c refers to the scenario with pion-condensation. The upper cooling curve is the same as in Fig. 4b (because low-mass stars have no inner cores), but the lower curve goes essentially higher (because the neutrino emission due to pion condensation is lower than due to direct Urca process). This scenario also explains all the sources and predicts the existence of cold stars, but not so cold as in the direct Urca case.

Fig. 4d is a similar plot for the scenario with kaon-condensation in the inner core. High-mass stars cool slower than in the previous scenarios, but all the data are still explained and a population of stars slightly colder than the observed ones is predicted.

Thus, all three scenarios are currently consistent with the observations, but predict different families of cold stars. Similar conclusions have been made by a number of authors (see, e.g., [9, 10] and references therein). Calculations show that the neutrino luminosity in the inner core should be $\approx 30 - 100$ times higher than the modified Urca luminosity to explain all the sources and leave no space for neutron stars colder than the observed ones. A discovery of very cold neutron stars would be crucial to firmly constrain the enhanced neutrino emission level.

Unfortunately, current observations give no reliable constraints on the position of the transition layer between slowly and fastly neutrino emitting regions in the core (on ρ_1 and ρ_2 or M_1 and M_2 in Fig. 3).

OTHER ASPECTS

We do not discuss in detail many aspects of the cooling theory but mention them here.

First, cooling can also be regulated by heat conduction in the neutron star crust. The thermal conductivity in the

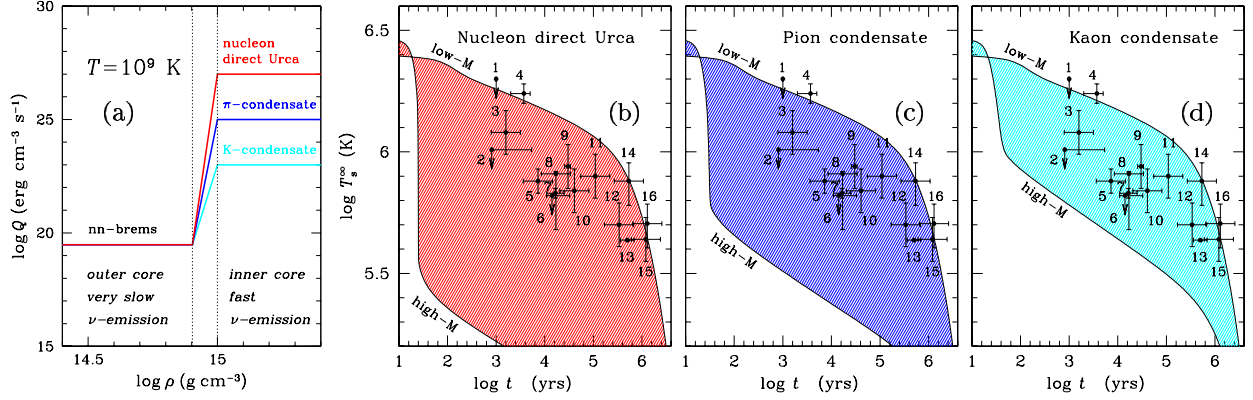


FIGURE 4. (a) Schematic density dependence of the neutrino emissivity in a neutron star core at $T = 10^9$ K assuming very slow neutrino emission in the outer core and three scenarios of fast emission in the inner core. (b), (c), and (d): Ranges of T_s^∞ (hatched) for the three types of fast emission compared with observations (see text for details).

crust can be affected by composition of the matter (e.g., by the presence of light elements in accreted surface layers) and magnetic fields (in the outer and inner crust; see, e.g., [9, 10, 56] and references therein). However, these cooling regulators are usually not as strong as neutrino emission from the core.

The effects of superfluidity can be more sophisticated than described above. Taking different EOSs [different direct Urca thresholds and $T_c(\rho)$ profiles] one can obtain a complicated zoo of families of cooling neutron stars [35, 57]. Neutrino emission due to Cooper pairing of baryons can affect the cooling in many ways as reviewed in Refs. [9, 10]. In particular, a moderately strong neutron superfluidity in the nucleon core can enhance the neutrino luminosity over the modified Urca level by a factor of ~ 30 , allowing one to explain the coldest observed sources by neutrino emission from the nucleon (non-exotic) core without invoking the direct Urca process. This leads to the scenario of “minimal cooling” [28, 29].

Cooling of neutron stars can be accompanied by some reheating associated, for instance, with the dissipation of differential rotation, deviation from beta-equilibrium or decay of magnetic fields in neutron star interiors. The magnetic reheating can be especially strong in magnetars (see below). Otherwise these reheating mechanisms are expected to be significant in old neutron stars (at the photon cooling stage). They are reviewed in [9, 10].

Finally, we have not discussed cooling of strange stars and neutron stars with quark cores. A comprehensive review is given in [10].

CONNECTIONS

With the spectacular progress of neutron star observations, cooling problem becomes more connected to other

aspects of neutron star physics.

First, cooling of isolated neutron stars is closely related to deep crustal heating of transiently accreting neutron stars in soft X-ray transients (SXTs). The matter accreted on a neutron star becomes eventually compressed by the weight of newly accreted material and sinks thus into the crust. Sinking into the deep crust is accompanied by nuclear transformations [58, 59, 60, 61] and associated heating of the entire star (1.5 – 2 MeV per one accreted nucleon). This deep crustal heating can keep thermally inertial stars warm [62] even in quiescent states of SXTs when accretion is switched off. It is likely responsible for thermal emission observed from some SXTs in quiescent states (e.g., Refs. [63, 64]).

One can distinguish two heating regimes. As a rule, accretion episodes are not too long (weeks-months) and do not violate isothermality of internal layers. These stars reach thermal quasiequilibrium determined by the mass accretion rates $\langle \dot{M} \rangle$ averaged over their global thermal relaxation time ($\sim 100 - 1000$ yr). Observations of these objects test essentially the same physics of neutron star interior as observations of cooling isolated neutron stars (e.g., [65, 66, 67]). The analysis of the data on SXTs [63, 64, 65] generally agrees with the above analysis of cooling neutron stars, but with important exceptions. The upper limits on T_s in quiescent states of the transiently accreting millisecond pulsar SAX J1808.4–3658 and possibly the SXT 1H 1905+000 are very low, giving an example of very cold neutron stars (missing so far in the data on cooling neutron stars). If true, these neutron stars undergo strong neutrino cooling via direct Urca process. However, in view of uncertainties associated with the deep crustal heating hypothesis, determination of T_s and $\langle \dot{M} \rangle$, these results should be taken with caution.

Some SXTs (KS 1731–260 and MXB 1659–29, e.g., [68]) show long accretion episodes (a few years and more) in which the crust may become much warmer than

the core. After accretion stops, the crust cools and thermally equilibrates with the core. This crust-core relaxation is reflected in the relaxation of the surface temperature $T_s(t)$, that is observed and can give important information on the crust and core physics [68, 69, 70, 71].

Cooling theory is also employed to study thermal states of magnetars. It seems one needs to introduce a reheating (most probably produced by strong magnetar magnetic fields) to explain high observed X-ray luminosities of magnetars. One can study quasistationary thermal states of magnetars (e.g., [72]) and afterburst relaxation (e.g., [73]).

In addition, cooling theory can be used to analyze the propagation of thermal waves in neutron stars during X-ray bursts and superbursts (e.g., [74, 75]) and to explore cooling of vibrating neutron stars [76].

Since the data are becoming numerous it is possible to combine cooling theory with neutron star statistics [77].

CONCLUSIONS

The theory of cooling neutron stars of ages $10^2 - 10^6$ yr mostly tests the neutrino emission properties of the neutron star core. Its main results are as follows.

(1) Neutrino emission in the outer core (i.e., in the core of a low-mass star) is a factor of 30–100 lower than the modified Urca emission in a nonsuperfluid star.

(2) Neutrino emission in the inner core (of a massive star) is at least a factor of 30–100 higher than the modified Urca emission. It can be enhanced by direct Urca process in nucleon/hyperon inner core or by the presence of pion or kaon condensate, or quark matter.

(3) The scenario with open direct Urca process predicts the existence (Fig. 4) of massive isolated neutron stars which are much colder than those observed now. In the scenario with pion condensate, the massive stars should be warmer (than those with open direct Urca) but colder than the observed ones. In the scenario with kaon condensate the massive stars should be even warmer but slightly colder than the observed sources. A discovery of cold cooling neutron stars would be crucial to constrain the level of enhanced neutrino emission in the inner core.

(4) Observations of cooling neutron stars can be analyzed together with observations of SXTs in quiescent states. The data on SXTs indicate the existence of very cold neutron stars (first of all, SAX J1808.4–3658) which cool via direct Urca process, but the data and interpretation require additional confirmation.

(5) A transition from slow neutrino emission in the outer core to enhanced emission in the inner core has to be smooth. Current observations of cooling neutron stars and SXTs do not constrain the parameters of this transition. A firm measurement of masses of cooling or accreting stars would help to impose such constraints.

(6) New observations and reliable practical theories of dense matter are vitally important to tune the cooling theory as an instrument for exploring physical properties of neutron star interiors and neutron star parameters. The tuning will imply a careful analysis of many cooling regulators.

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REFERENCES

1. P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, *Neutron Stars. I. Equation of State and Structure*, Springer, New York, 2007.
2. J. M. Lattimer and M. Prakash, *Phys. Rep.*, **442**, 109–165 (2007).
3. R. Stabler, Ph. D. thesis (Cornell University, 1960).
4. H.-Y. Chiu, *Ann. Phys.*, **26**, 364–410 (1964).
5. D. C. Morton, *Nature*, **201**, 1308–1309 (1964).
6. H.-Y. Chiu and E. E. Salpeter, *Phys. Rev. Lett.*, **12**, 413–415 (1964).
7. J. N. Bahcall and R. A. Wolf, *Phys. Rev.*, **140**, B1452–B1466 (1965).
8. S. Tsuruta and A. G. W. Cameron, *Canad. J. Phys.*, **44**, 1863–1894 (1966).
9. D. G. Yakovlev and C. J. Pethick, *Annu. Rev. Astron. Astrophys.*, **42**, 169–210 (2004).
10. D. Page, U. Geppert, and F. Weber, *Nucl. Phys. A*, **777**, 497–530 (2006).
11. N. Glendenning, *Compact Stars. Nuclear Physics, Particle Physics and General Relativity*, Springer, New York, 1996.
12. J. A. Pons, J. A. Miralles, M. Prakash, and J. M. Lattimer, *Astrophys. J.*, **553**, 382–393 (2001).
13. K. S. Thorne, *Astrophys. J.*, **212**, 825–831 (1977).
14. E. H. Gudmundsson, C. J. Pethick, and R. I. Epstein, *Astrophys. J.*, **272**, 286–300 (1983).
15. A. Y. Potekhin and D. G. Yakovlev, *Astron. Astrophys.*, **374**, 213–226 (2001).
16. J. M. Lattimer, K. A. Van Riper, M. Prakash, and M. Prakash, *Astrophys. J.*, **425**, 802–813 (1994).
17. O. Y. Gnedin, D. G. Yakovlev, and A. Y. Potekhin, *Mon. Not. Roy. Astron. Soc.*, **324**, 725–736 (2001).
18. C. J. Pethick, *Rev. Mod. Phys.*, **64**, 1133–1140 (1992).
19. D. G. Yakovlev, K. P. Levenfish, and Yu. A. Shibano, *Physics – Uspekhi*, **42**, 737–778 (1999).
20. D. G. Yakovlev, A. D. Kaminker, O. Y. Gnedin, and P. Haensel, *Phys. Rep.*, **354**, 1–155 (2001).
21. J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, *Phys. Rev. Lett.*, **66**, 2701–2704 (1991).
22. M. Prakash, M. Prakash, J. M. Lattimer, and C. J. Pethick, *Astrophys. J.*, **390**, L77–L80 (1992).

23. U. Lombardo and H.-J. Schulze, “Superfluidity in neutron star matter,” in *Physics of Neutron Star Interiors*, edited by D. Blaschke, N. Glendenning, and A. Sedrakian, Springer, Berlin, 2001, pp. 30–53.
24. D. Page and J. H. Applegate, *Astrophys. J.*, **394**, L17–L20 (1992).
25. R. Tamagaki and T. Takatsuka, *Progr. Theor. Phys.*, **117**, 861–901 (2007).
26. M. Alford, K. Rajagopal, and F. Wilczek, *Phys. Lett. B*, **422**, 247–256 (1998).
27. E. G. Flowers, M. Ruderman, and P. G. Sutherland, *Astrophys. J.*, **205**, 541–544 (1976).
28. D. Page, J. M. Lattimer, M. Prakash, and A. W. Steiner, *Astrophys. J. Suppl. Ser.*, **155**, 623–650 (2004).
29. M. E. Gusakov, A. D. Kaminker, D. G. Yakovlev, and O. Y. Gnedin, *Astron. Astrophys.*, **423**, 1063–1072 (2004).
30. D. A. Baiko, P. Haensel, and D. G. Yakovlev, *Astron. Astrophys.*, **374**, 151–163 (2001).
31. P. S. Shternin and D. G. Yakovlev, *Phys. Rev. D*, **75**, 103004 (2007).
32. V. E. Zavlin, “Thermal emission from isolated neutron stars: theoretical and observational aspects,” *Springer Lecture Notes*, 2007, in press [astro-ph/0702426]
33. A. de Luca, this volume.
34. D. Kaplan, this volume.
35. A. D. Kaminker, M. E. Gusakov, D. G. Yakovlev, and O. Y. Gnedin, *Mon. Not. Roy. Astron. Soc.*, **365**, 1300–1308 (2006).
36. D. G. Yakovlev, O. Y. Gnedin, A. D. Kaminker, K. P. Levenfish, and A. Y. Potekhin, *Adv. Space Res.*, **33**, 523–530 (2004).
37. M. C. Weisskopf, S. L. O’Dell, F. Paerels, R. F. Elsner, W. Becker, A. F. Tennant, and D. A. Swartz, *Astrophys. J.*, **601**, 1050–1057 (2004).
38. P. Slane, D. J. Helfand, E. van der Swaluw, and S. S. Murray, *Astrophys. J.*, **616**, 403–413 (2004).
39. V. E. Zavlin, J. Trümper, and G. G. Pavlov, *Astrophys. J.*, **525**, 959–967 (1999).
40. V. E. Zavlin, *Astrophys. J.*, **665**, L143–L146 (2007).
41. J. P. Halpern, E. V. Gotthelf, F. Camilo, D. J. Helfand, and S. M. Ransom, *Astrophys. J.*, **612**, 398–407 (2004).
42. G. G. Pavlov, V. E. Zavlin, D. Sanwal, V. Burwitz, and G. P. Garmire, *Astrophys. J.*, **552**, L129–L133 (2001).
43. K. E. McGowan, S. Zane, M. Cropper, J. A. Kennea, F. A. Córdova, C. Ho, T. Sasseen, and W. T. Vestrand, *Astrophys. J.*, **600**, 343–350 (2004).
44. V. E. Zavlin and G. G. Pavlov, *Mem. Soc. Astron. Ital.*, **75**, 458–463 (2004).
45. A. Possenti, S. Mereghetti, and M. Colpi, *Astron. Astrophys.*, **313**, 565–570 (1996).
46. O. Y. Kargaltsev, G. G. Pavlov, V. E. Zavlin, and R. W. Romani, *Astrophys. J.*, **625**, 307–323 (2005).
47. W. C. G. Ho, D. L. Kaplan, P. Chang, M. van Adelsberg, and A. Y. Potekhin, *Mon. Not. Roy. Astron. Soc.*, **375**, 821–830 (2007).
48. G. G. Pavlov and V. E. Zavlin, “Thermal radiation from cooling neutron stars,” in *Texas in Tuscany. XXI Texas Symposium on Relativistic Astrophysics*, edited by R. Bandiera, R. Maolino, and F. Mannucci, Singapore, World Scientific, 2003, pp. 319–328.
49. C. Motch, V. E. Zavlin, and F. Haberl, *Astron. Astrophys.*, **408**, 323–330 (2003).
50. K. Mori and W. C. G. Ho, *Mon. Not. Roy. Astron. Soc.*, **377**, 905–919 (2007).
51. M. Prakash, T. L. Ainsworth, and J. M. Lattimer J.M., *Phys. Rev. Lett.*, **61**, 2518–2521 (1988).
52. A. D. Kaminker, D. G. Yakovlev, and O. Y. Gnedin, *Astron. Astrophys.*, **383**, 1076–1087 (2002).
53. M. E. Gusakov, A. D. Kaminker, D. G. Yakovlev, and O. Y. Gnedin, *Astronomy Letters*, **30**, 759–771 (2004).
54. Ch. Schaab, F. Weber, M. K. Weigel, and N. K. Glendenning, *Nucl. Phys. A*, **605**, 531–565 (1996).
55. D. A. Baiko and D. G. Yakovlev, *Astron. Astrophys.*, **342**, 192–200 (1999).
56. U. Geppert, M. Küker, and D. Page, *Astron. Astrophys.*, **457**, 937–947 (2006).
57. M. E. Gusakov, A. D. Kaminker, D. G. Yakovlev, and O. Y. Gnedin, *Mon. Not. Roy. Astron. Soc.*, **363**, 555–562 (2005).
58. P. Haensel and J. L. Zdunik, *Astron. Astrophys.*, **227**, 431–36 (1990).
59. P. Haensel and J. L. Zdunik, *Astron. Astrophys.*, **404**, L33–L36 (2003).
60. S. Gupta, E. F. Brown, H. Schatz, P. Möller, and K.-L. Kratz, *Astron. Astrophys.*, **662**, 1188–1197 (2007).
61. P. Haensel and J. L. Zdunik, *Astron. Astrophys.*, 2007, submitted [arXiv:0708.3996].
62. E. F. Brown, L. Bildsten, and R. E. Rutledge, *Astrophys. J.*, **504**, L95–L98 (1998).
63. C. O. Heinke, C. J. Deloye, P. G. Jonker, R. E. Taam, and R. Wijnands, this volume [arXiv:0710.1552].
64. P. Jonker, this volume.
65. K. P. Levenfish and P. Haensel, *Astrophys. Space Sci.*, **308**, 457–465 (2007).
66. D. G. Yakovlev, K. P. Levenfish, and P. Haensel, *Astron. Astrophys.*, **407**, 265–271 (2003).
67. D. G. Yakovlev, K. P. Levenfish, A. Y. Potekhin, O. Y. Gnedin, and G. Chabrier, *Astron. Astrophys.*, **417**, 169–179 (2004).
68. E. M. Cackett, R. Wijnands, M. Linares, J. M. Miller, J. Homan, and W. H. G. Lewin, *Mon. Not. Roy. Astron. Soc.*, **372**, 479–488 (2007).
69. G. Ushomirsky and R. E. Rutledge *Mon. Not. Roy. Astron. Soc.*, **325**, 1157–1166 (2001).
70. R. E. Rutledge, L. Bildsten, E. F. Brown, G. G. Pavlov, V. E. Zavlin, and G. Ushomirsky, *Astrophys. J.*, **580**, 413–422 (2002).
71. P. S. Shternin, D. G. Yakovlev, P. Haensel, and A. Y. Potekhin, *Mon. Not. Roy. Astron. Soc.*, **382**, L43–L47 (2007).
72. A. D. Kaminker, D. G. Yakovlev, A. Y. Potekhin, N. Shibazaki, P. S. Shternin, and O. Y. Gnedin, *Mon. Not. Roy. Astron. Soc.*, **371**, 477–483 (2006).
73. Y. Lubarsky, D. Eichler, and C. Thompson, *Astrophys. J.*, **580**, L69–L72 (2002).
74. M. Y. Fujimoto, T. Hanawa, I. Iben, Jr., and M. B. Richardson, *Astrophys. J.*, **278**, 813–824 (1984).
75. A. Cumming, J. Macbeth, J. J. M. in’t Zand, D. Page, *Astrophys. J.*, **646**, 429–451 (2006).
76. M. E. Gusakov, D. G. Yakovlev, and O. Y. Gnedin, *Mon. Not. Roy. Astron. Soc.*, **361**, 1415–1424 (2005).
77. S. Popov, H. Grigorian, R. Turolla, and D. Blaschke, *Astron. Astrophys.*, **448**, 327–334, 2006.